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Effects of temperature gradients and sheath power transmission on Langmuir probes

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Abstract

Atomic physics effects, which can be critical for understanding and controlling plasmas near the material surfaces in magnetic fusion devices and technical plasmas, are often sensitive functions of the electron temperature, making accurate temperature measurements important. Questions have been raised whether the temperature extracted from Langmuir probes properly accounts for temperature profile and ohmic heating effects. Careful attention must be given to the definition of the effective temperature of a non-Maxwellian distribution, and an effort made to discuss self-consistent profiles. Also, depending on the upstream boundary conditions imposed, the instantaneous plasma parameters can depend on the bias voltage applied. While significant errors seem possible, quantitative understanding will require more detailed and realistic models.

Keywords: Langmuir probe; Velocity distribution

1. The meaning of 'temperature'

A non-Maxwellian electron velocity distribution function (vdf) resulting from temperature gradients was considered by Wesson [1], who concluded that the sheath can easily be dominated by the distant temperature. To arrive at this conclusion, he defined an 'effective' temperature as the uniform temperature which, for a given current, would give the same sheath potential. More precisely, it can be found implicitly from

$$j_e = ne(T_{\text{Wesson}}/2\pi m_e)^{1/2} \exp(V/T_{\text{Wesson}}), \quad (1)$$

where $j_e = j - j_i$ is the electron current density and V is the voltage of the probe relative to the plasma potential. This definition is a function of voltage and requires knowledge of the plasma potential and density. It can thus be used to summarize a theoretical solution, but it cannot be directly related to an experimental I–V characteristic.

Of the minimal three I–V pairs required for a probe measurement, one is usually used to measure the ion saturation current, and the others are placed near floating. In the limit that these two coincide, we have information on the derivative as well as the value of the current, allowing us to define a temperature in terms of the slope of the I–V characteristic:

$$T_{\text{slope}} = \frac{j_e}{(\mathrm{d}j_e/dV)}. \quad (2)$$

This definition is interesting for its similarity to single and triple probe practice and its accessibility to linear theories. If we move our third point from the ion branch to the neighborhood of floating, the measurement will be completely local in voltage. In the limit of coincidence, the third point allows us to measure the curvature of the I–V, so we can define another effective temperature,

$$T_{\text{curv}} = \frac{(\mathrm{d}j_e/\mathrm{d}V)}{(\mathrm{d}^2 j_e/dV^2)}. \quad (3)$$

This definition can be experimentally realized [2] by applying a small sinusoidal voltage and taking the ratio of the fundamental and first harmonic of the current signal.

To generate a non-Maxwellian vdf for test purposes, we follow Wesson's procedure of assuming a tempera-

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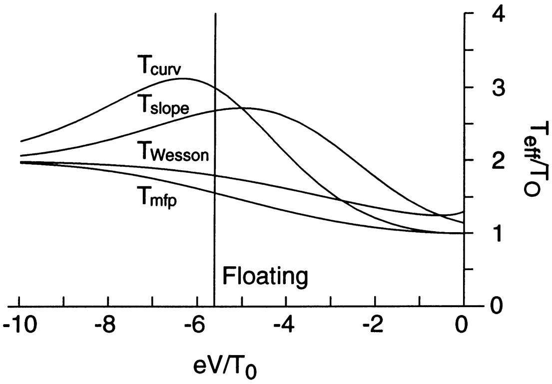

Fig. 1. Comparison of four definitions of 'effective' temperature, when $T_\infty/T_0 = 2$ and $\ell/\lambda_0 = 200$. At large negative voltages, only the tail electrons contribute, and all measures approach T_∞. At plasma potential ($V = 0$), the bulk electrons predominate, so all measures are near T_0. At floating potential (the vertical line), the mixture of temperatures leads to $T_{\rm slope}$and $T_{\rm curv} > T_\infty$.

ture profile of the form $T(x) = T_0 + (T_\infty - T_0)\tanh(x/\ell)$ and building up a vdf by taking the value at velocity v to be that of a Maxwellian with temperature $T(\lambda(v))$, where $\lambda(v) = (v/v_0)^4\lambda_0$ is the mean free path of electrons with velocity v, $\lambda_0 = 0.129 v_0\tau_{\rm e}$ and $v_0 = (T_0/m_{\rm e})^{1/2}$. In Fig. 1 we have plotted the temperatures as a function of voltage calculated with (1)–(3). The vertical line, where $j_{\rm e} = ne(T_\infty/2m_{\rm p})^{1/2}$, approximates the floating potential for cold deuterium ions. In addition we have plotted $T_{\rm mfp}$, the temperature a distance from the surface equal to one mean free path for electrons that just pass the Debye sheath, and note that it is significantly smaller than all the derived temperatures, so the colloquial statement that Langmuir probes "measure the temperature a tail mean free path away" is not very accurate. More disturbing is the behavior of $T_{\rm slope}$ and $T_{\rm curv}$, which are greater than $2.7T_0$ near floating, although the highest temperature in the system is only $2T_0$. This result is surprising, but can also be analytically derived for some simple cases, such as a step function temperature profile. It underscores the importance of careful consideration of what one chooses to call the 'effective temperature'. We consider $T_{\rm slope}$ at floating to be a good compromise between experimental relevance, theoretical convenience, and transparency.

Wesson's assumption that the mean free path scales with v^4 is only true if Coulomb collisions dominate, in which case the electron heat loss will occur primarily through classical conduction and convection. The convective heat flux upstream, found by integrating a shifted Maxwellian vdf, is $(5/2)T_\infty\Gamma_{\rm e}$. The heat flux through the Debye sheath, found by integrating a truncated Maxwellian, is $5T_0\Gamma_{\rm e}$ (for a surface near floating). If there are no sources or sinks of heat, we can equate these two fluxes, leading to $T_\infty \approx 2T_0$. At the sheath, the conductive heat flux must make up for the half of the convective heat flux lost, $\kappa_0({\rm d}T/{\rm d}x)_0 = (5/2)T_0\Gamma_{\rm e}$. With classical conductivity given by $\kappa = 3.2nT\tau_{\rm e}/m_{\rm e}$, we find the scale length $l = (T_\infty - T_0)({\rm d}T/{\rm d}x)_0^{-1} = 410\lambda_0$, so we expect to be in a regime where the result depends on the exact definition of temperature used and other details of the model. The velocity distribution as calculated with a particle-in-cell simulation modeling electron–electron Coulomb collisions also found a cool bulk and warm tail, but the transition was in some way softer, so that the slope temperature lay between T_0 and T_∞, suggesting that the questions addressed here are not quite as severe as they appear.

For many interesting conditions, some process other than Coulomb collisions will be dominant, whether radiation, ionization, radiative recombination, or three-body recombination. In particular, the physics of the electron–ion collisions resulting in a resistive voltage drop is essentially the same as that of the electron–electron collisions determining the heat conduction. Consequently, if Wesson's model is applicable, then bulk resistivity will also affect the I–V characteristic. We expect consideration of the definition of the temperature to retain its importance in models which embody other physics.

2. Effect of probe bias on plasma parameters

We now put aside temperature profile effects to consider changes in plasma parameters associated with the probe bias. Yu and Wesson [3] considered whether a probe will measure a higher temperature because it heats the plasma with its own current. This can be a problem, but only for current densities which are so high that parallel resistance is also a problem. If the parallel path of the current is $L_\parallel$, then the additional heat flux due to ohmic heating is $\eta j^2 L_\parallel$. If this heating results in a temperature rise ΔT, then the additional heat flux through the sheath, assuming the particle flux and sheath power transmission factor are not changed, will be about $5\Delta T\Gamma_i$. Equating the two heat fluxes, we find

$$\frac{\Delta T}{T} = \frac{\eta j_{\rm sat} L_\parallel}{T/e}\,\frac{(j/j_{\rm sat})^2}{5}, \tag{4}$$

with $j_{\rm sat} = e\Gamma_i$. In practice, only probe measurements with $|j| \lesssim j_{\rm sat}$ are used to determine the temperature, so the second factor on the right is small. In order for the fractional increase in temperature to be significant, the first factor on the right must be large, but this is the ratio of the voltage dropped in the bulk plasma to that dropped in the sheath. In other words, as T decreases or $L_\parallel$ grows, the voltage drop in the bulk plasma invalidates the traditional temperature measurement before the ohmic heating does.

There are, however, additional heating and cooling terms, not identified by Yu and Wesson, which have more serious effects. When interpreting the I–V characteristic of a Langmuir probe, it is usually assumed that the plasma parameters remain constant as the probe voltage is scanned. Over a region without particle, momentum, or energy sources, the current density j, the ion flux Γ, the dynamic pressure $P = n\left(2T + \frac{1}{2}m_i v^2\right)$, (assuming $T_i = T_e$ everywhere), and the heat flux Q are all spatially uniform, but the values these parameters take on may depend on the interplay of the sources with the boundary conditions at the surface. We will illustrate the breadth of the possibilities by considering combinations where two of Γ, P, and Q are taken as given (independent of the voltage of the surface relative to the plasma), and the third one free.

The heat flux from the plasma, including the effects of secondary electron emission with coefficient γ, can be written as [4]

$$Q/T\Gamma = \delta(\psi) = 2 + (2(1-\gamma)^{-1} - \psi)\exp(\psi - \psi_{\rm fl}),$$

where $\psi = eV/T(V)$. We assume that the velocity at the sheath edge is given by $v = 2\sqrt{T/m}$, the marginal form of the Bohm criterion with $\gamma_i = 3$, so that $\Gamma = 2n_0(T_0/m_i)^{1/2}$, and $P = 4n_0T_0$. The solutions for T and j are plotted as functions of $e(V - V_{\rm fl}) = \psi T(\psi) - \psi_{\rm fl}T(\psi_{\rm fl})$ in Fig. 2. A useful summary of this information is the saturation flux, $\Gamma_{\rm sat} = \lim_{\psi\to-\infty}\Gamma$ and the slope temperature defined in (2), which may differ from $\Gamma_{\rm fl}$ and $T_{\rm fl}$, the flux and temperature in the unperturbed plasma. We now consider the effect of various upstream boundary conditions.

If we hold Γ and P constant and let Q be a function of the voltage, then $T(\psi) = (4/9)(P^2/m\Gamma^2) = {\rm const}$. This yields the classical I–V characteristic with $\Gamma_{\rm sat} = \Gamma_{\rm fl}$ and $T_{\rm slope} = T_{\rm fl}$. Now we hold Γ and Q constant and let P be a function of the voltage. Of course, we have again $\Gamma_{\rm sat} = \Gamma_{\rm fl}$. We find $T_{\rm slope}/T_{\rm fl} = 1 + (-\psi_{\rm fl})[{\rm d}\ln\delta/{\rm d}\psi]_{\rm fl}$. For $m_i/m_{\rm p} = 2$ and $\gamma = 0.4$, this reduces to $T_{\rm slope}/T_{\rm fl} = 2.4$. Finally we hold P and Q fixed and allow Γ to vary. We find $T(\psi) = (9/4)(mQ/P^2)\delta^{-2}(\psi)$ and $\Gamma(\psi) = (4/9)(P^2/mQ)\delta(\psi)$. For $m_i/m_{\rm p} = 2$ and $\gamma = 0.4$, this leads to $\Gamma_{\rm sat}/\Gamma_{\rm fl} = 0.26$ and $T_{\rm slope}/T_{\rm fl} = 1.00$. So, for these particular values, the apparent temperature is correct, but the flux (or density) is four times smaller than the unperturbed value. This result is not quite acceptable. It turns out that the 'ion' current has a maximum near $V = -10T_{\rm slope}/e$, which in practice would be interpreted as the saturation current. If that is done, then $\Gamma_{\rm sat}/\Gamma_{\rm fl} = 0.37$ and $T_{\rm slope}/T_{\rm fl} = 1.42$.

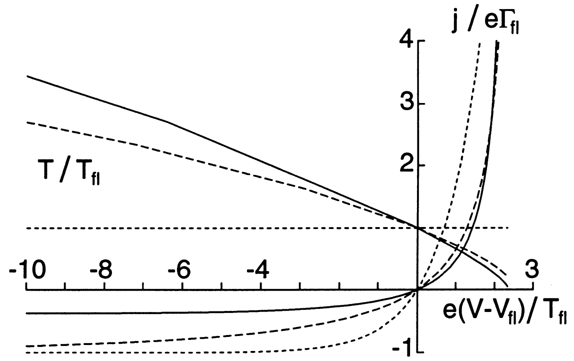

Fig. 2. The temperature of the plasma in front of a biased surface and the current to that surface for three boundary conditions: P and Γ fixed (dotted line), Γ and Q fixed (dashed line), and P and Q fixed (solid line).

In conclusion, we have shown that statements about the accuracy of Langmuir probes can only be made if I–V characteristics are actually investigated. 'Effective' parameters, derived, for example, from the vdf, can be misleading. We have investigated various effects which can modify the characteristics and lead to errors in the interpretation on the order of a factor of 2. We are not yet able to say how to correct probe measurements for these effects, or even whether they actually exist in situations of interest.

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